

# GCSE MATHS

## COMPLETE FORMULA SHEET

All 5 Strands · Foundation & Higher · AQA, Edexcel & OCR

NUMBER

ALGEBRA

RATIO & RATES

GEOMETRY

STATISTICS

**90+**

FORMULAS

**5**

STRANDS

**Both**

TIERS

**3**

EXAM BOARDS

**Free**

RESOURCE

**F & H**

= Foundation & Higher

**Higher**

= Higher tier only

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| NUMBER                         |   |        |
|--------------------------------|---|--------|
| Fractions — Adding/Subtracting | Find common denominator, then add/subtract numerators                     | F & H  |
| Fractions — Multiplying        | $a/b \times c/d = ac / bd$  | F & H  |
| Fractions — Dividing           | $a/b \div c/d = a/b \times d/c = ad / bc$                                 | F & H  |
| Percentage of an Amount        | % of amount = (percentage $\div$ 100) x amount                            | F & H  |
| Percentage Change              | % change = (change $\div$ original) x 100                                 | F & H  |
| Percentage Increase/Decrease   | new value = original x (1 + r/100)<br><i>decrease: use 1 - r/100</i>      | F & H  |
| Reverse Percentage             | original = final amount $\div$ (1 $\pm$ r/100)                            | F & H  |
| Simple Interest                | Interest = (P x R x T) / 100<br><i>P = principal, R = rate, T = time</i>  | F & H  |
| Compound Interest              | $A = P (1 + r/100)^n$<br><i>P = principal, r = rate, n = years</i>        | F & H  |
| Depreciation / Decay           | $A = P (1 - r/100)^n$   | F & H  |
| HCF x LCM Rule                 | $HCF(a,b) \times LCM(a,b) = a \times b$                                   | F & H  |
| Prime Factorisation            | Write as product of primes, e.g.<br>$60 = 2^2 \times 3 \times 5$          | F & H  |
| Index Law — Multiply           | $a^m \times a^n = a^{(m+n)}$  | F & H  |
| Index Law — Divide             | $a^m \div a^n = a^{(m-n)}$  | F & H  |
| Index Law — Power of Power     | $(a^m)^n = a^{(m \times n)}$  | F & H  |
| Index Law — Zero Power         | $a^0 = 1$ (for any $a \neq 0$ )   | F & H  |
| Negative Index                 | $a^{-n} = 1 / a^n$  | F & H  |
| Fractional Index               | $a^{(1/n)} =$ nth root of a<br>$a^{(m/n)} =$ (nth root of a) <sup>m</sup> | Higher |
| Standard Form                  | $A \times 10^n$ where $1 \leq A < 10$                                     | F & H  |
| Surd — Simplify                | $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$                                    | Higher |
| Surd — Rationalise Denominator | $a / \sqrt{b} = (a \times \sqrt{b}) / b$                                  | Higher |

## NUMBER

|                              |   |        |
|------------------------------|---|--------|
| Surd — Difference of Squares | $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$  | Higher |
| Upper & Lower Bounds         | $UB = \text{value} + 0.5 \times \text{precision}$<br>$LB = \text{value} - 0.5 \times \text{precision}$                | Higher |
| Bounds — Max/Min Results     | $\max(a+b) = UB(a) + UB(b)$ ;<br>$\min(a-b) = LB(a) - UB(b)$<br><small><math>\max(a/b) = UB(a) / LB(b)</math></small> | Higher |

## ALGEBRA

|                               |   |        |
|-------------------------------|---|--------|
| Expanding — Single Bracket    | $a(b + c) = ab + ac$  | F & H  |
| Expanding — Double Brackets   | $(a+b)(c+d) = ac + ad + bc + bd$  | F & H  |
| Difference of Two Squares     | $a^2 - b^2 = (a+b)(a-b)$  | F & H  |
| Perfect Square                | $(a+b)^2 = a^2 + 2ab + b^2$<br>$(a-b)^2 = a^2 - 2ab + b^2$                                | F & H  |
| Gradient of a Line            | $m = (y_2 - y_1) / (x_2 - x_1)$   | F & H  |
| Equation of a Straight Line   | $y = mx + c$<br><i>m = gradient, c = y-intercept</i>                                      | F & H  |
| Perpendicular Lines           | $m_1 \times m_2 = -1$ (gradients multiply to -1)  | Higher |
| Nth Term — Linear Sequence    | $\text{nth term} = a + (n-1)d = dn + (a-d)$<br><i>d = common difference, a = 1st term</i> | F & H  |
| Nth Term — Quadratic Sequence | $\text{nth term} = an^2 + bn + c$<br><i>a = 2nd difference <math>\div</math> 2</i>        | Higher |
| Quadratic Formula             | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$<br><i>for <math>ax^2 + bx + c = 0</math></i>     | F & H  |
| Completing the Square         | $ax^2 + bx + c = a(x + b/2a)^2 - (b/2a)^2 + c$  | Higher |
| Discriminant                  | $b^2 - 4ac > 0$ two real roots<br>$= 0$ one root; $< 0$ no real roots                     | Higher |
| Algebraic Fractions           | Same rules as numeric fractions — factorise first   | Higher |
| Simultaneous — Elimination    | Multiply to match coefficients, then add or subtract                                      | F & H  |
| Simultaneous — Substitution   | Rearrange one equation, substitute into the other   | F & H  |
| Direct Proportion             | $y = kx$ (y proportional to x)<br><i>find k using given values</i>                        | F & H  |
| Inverse Proportion            | $y = k/x$ (y inversely proportional to x)   | F & H  |
| Function Notation             | $f(x)$ means output of function f for input x   | Higher |
| Composite Function            | $fg(x) = f(g(x))$ — apply g first, then f   | Higher |
| Inverse Function              | $f^{-1}(x)$ : swap x and y, then rearrange for y  | Higher |

## ALGEBRA

|                       |   |        |
|-----------------------|---|--------|
| Equation of a Circle  | $x^2 + y^2 = r^2$ (centre at origin)<br>$(x-a)^2 + (y-b)^2 = r^2$ (centre $a,b$ )                                     | Higher |
| Graph Transformations | $f(x) + a$ = shift up $a$ ; $f(x+a)$ = shift left $a$<br>$af(x)$ = stretch vertically; $f(ax)$ = stretch horizontally | Higher |
| Iteration             | $x_{(n+1)} = f(x_n)$ — substitute repeatedly to converge  | Higher |
| Algebraic Proof       | Use general expressions: even = $2n$ , odd = $2n+1$<br>consecutive integers: $n, n+1, n+2 \dots$                      | Higher |

## RATIO, PROPORTION &amp; RATES OF CHANGE

|                           |  |        |
|---------------------------|--|--------|
| Ratio — Sharing           | total $\div$ sum of parts $\times$ each share                                    | F & H  |
| Ratio — Simplifying       | Divide all parts by HCF  | F & H  |
| Ratio — Unitary Method    | Find value of 1 part, then multiply  | F & H  |
| Speed                     | Speed = Distance $\div$ Time   | F & H  |
| Distance                  | Distance = Speed $\times$ Time   | F & H  |
| Time                      | Time = Distance $\div$ Speed   | F & H  |
| Density                   | Density = Mass $\div$ Volume   | F & H  |
| Mass                      | Mass = Density $\times$ Volume   | F & H  |
| Volume (from density)     | Volume = Mass $\div$ Density   | F & H  |
| Pressure                  | Pressure = Force $\div$ Area   | F & H  |
| Force                     | Force = Pressure $\times$ Area   | F & H  |
| Area (from pressure)      | Area = Force $\div$ Pressure   | F & H  |
| Percentage Increase       | new = original $\times (1 + r/100)$  | F & H  |
| Percentage Decrease       | new = original $\times (1 - r/100)$  | F & H  |
| Reverse Percentage        | original = final $\div$ multiplier<br>e.g. after 20% increase: $\div 1.2$        | F & H  |
| Compound Interest         | $A = P(1 + r/100)^n$   | F & H  |
| Depreciation              | $A = P(1 - r/100)^n$   | F & H  |
| Scale Factor — Length     | SF = new length $\div$ original length   | F & H  |
| Scale Factor — Area       | Area SF = (length SF) <sup>2</sup>   | Higher |
| Scale Factor — Volume     | Volume SF = (length SF) <sup>3</sup>   | Higher |
| Unit Conversions — Area   | $1 \text{ m}^2 = 10,000 \text{ cm}^2$ ; $1 \text{ km}^2 = 1,000,000 \text{ m}^2$ | F & H  |
| Unit Conversions — Volume | $1 \text{ m}^3 = 1,000,000 \text{ cm}^3$ ; 1 litre = 1000 $\text{cm}^3$          | F & H  |

## RATIO, PROPORTION &amp; RATES OF CHANGE

Growth &amp; Decay

amount = initial x  
(multiplier)<sup>time</sup>

Higher

## GEOMETRY &amp; MEASURES

|                         |   |       |
|-------------------------|---|-------|
| Area — Rectangle        | $A = \text{length} \times \text{width}$   | F & H |
| Area — Triangle         | $A = \frac{1}{2} \times \text{base} \times \text{height}$   | F & H |
| Area — Parallelogram    | $A = \text{base} \times \text{perpendicular height}$  | F & H |
| Area — Trapezium        | $A = \frac{1}{2} (a + b) \times h$<br><i>a, b = parallel sides, h = height</i>  | F & H |
| Area — Circle           | $A = \pi \times r^2$  | F & H |
| Circumference           | $C = 2 \times \pi \times r = \pi \times d$  | F & H |
| Arc Length              | $\text{arc} = \left(\frac{\text{theta}}{360}\right) \times 2 \times \pi \times r$   | F & H |
| Sector Area             | $\text{sector} = \left(\frac{\text{theta}}{360}\right) \times \pi \times r^2$   | F & H |
| Volume — Prism          | $V = \text{cross-section area} \times \text{length}$  | F & H |
| Volume — Cylinder       | $V = \pi \times r^2 \times h$   | F & H |
| Volume — Cone           | $V = \frac{1}{3} \times \pi \times r^2 \times h$  | F & H |
| Volume — Sphere         | $V = \frac{4}{3} \times \pi \times r^3$   | F & H |
| Volume — Pyramid        | $V = \frac{1}{3} \times \text{base area} \times \text{height}$  | F & H |
| Surface Area — Cylinder | $SA = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h$   | F & H |
| Surface Area — Cone     | $SA = \pi \times r \times l + \pi \times r^2$<br><i>l = slant height</i>  | F & H |
| Surface Area — Sphere   | $SA = 4 \times \pi \times r^2$  | F & H |
| Pythagoras' Theorem     | $a^2 + b^2 = c^2$<br><i>c = hypotenuse (longest side)</i>   | F & H |
| Trigonometry — SOH      | $\sin(\text{theta}) = \frac{\text{Opposite}}{\text{Hypotenuse}}$  | F & H |
| Trigonometry — CAH      | $\cos(\text{theta}) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$  | F & H |
| Trigonometry — TOA      | $\tan(\text{theta}) = \frac{\text{Opposite}}{\text{Adjacent}}$  | F & H |
| Exact Trig Values       | $\sin 30 = \frac{1}{2}, \sin 45 = \frac{1}{\sqrt{2}},$<br>$\sin 60 = \frac{\sqrt{3}}{2}$<br><i><math>\cos 30 = \frac{\sqrt{3}}{2}, \cos 45 = \frac{1}{\sqrt{2}},</math><br/><math>\cos 60 = \frac{1}{2}; \tan 45 = 1, \tan 60 = \sqrt{3}</math></i> | F & H |

| GEOMETRY & MEASURES             |   |
|---------------------------------|---|
| Sine Rule                       | $a / \sin A = b / \sin B = c / \sin C$<br>use when: 2 angles + 1 side OR 2 sides + non-included angle<br>Higher |
| Cosine Rule — Find Side         | $a^2 = b^2 + c^2 - 2bc \cos(A)$<br>use when: 2 sides + included angle<br>Higher                                 |
| Cosine Rule — Find Angle        | $\cos(A) = (b^2 + c^2 - a^2) / (2bc)$<br>use when: all 3 sides known<br>Higher                                  |
| Area of Triangle (Trig)         | $\text{Area} = 1/2 \times a \times b \times \sin(C)$<br>use when: 2 sides + included angle<br>Higher            |
| 3D Pythagoras                   | $d = \sqrt{x^2 + y^2 + z^2}$<br>Higher  |
| Angles in a Triangle            | Sum of angles = 180° F & H  |
| Angles on a Straight Line       | Sum of angles = 180° F & H  |
| Angles at a Point               | Sum of angles = 360° F & H  |
| Angles in a Polygon             | Sum of interior angles = $(n - 2) \times 180^\circ$ F & H   |
| Exterior Angle of Polygon       | Each exterior angle = $360^\circ \div n$<br>exterior + interior = 180° F & H                                    |
| Circle Theorem 1                | Angle at centre = 2 x angle at circumference (same arc)<br>Higher   |
| Circle Theorem 2                | Angle in a semicircle = 90° Higher  |
| Circle Theorem 3                | Angles in same segment are equal Higher   |
| Circle Theorem 4                | Opposite angles in cyclic quadrilateral = 180° Higher   |
| Circle Theorem 5                | Tangent is perpendicular to radius Higher   |
| Circle Theorem 6                | Two tangents from external point are equal length Higher  |
| Circle Theorem 7                | Alternate segment theorem: angle in alt segment = tangent Higher  |
| Vectors — Addition              | $AB + BC = AC$ Higher   |
| Vectors — Scalar Multiplication | $k \times \text{vector } a = \text{stretch by factor } k$ Higher  |
| Vectors — Midpoint              | midpoint $M = OA + 1/2 AB$ Higher   |
| Bearings                        | Measured clockwise from North (000° to 360°) F & H  |

| STATISTICS & PROBABILITY      |   |
|-------------------------------|---|
| Mean — Ungrouped Data         | $\text{mean} = \text{sum of values} \div \text{number of values } (\Sigma x / n)$<br>F & H  |
| Mean — Grouped Data           | $\text{mean} = \Sigma(f \times \text{midpoint}) \div \Sigma f$<br>$f = \text{frequency, midpoint} = \text{class centre}$<br>F & H |
| Median                        | Middle value when ordered;<br>position = $(n+1) / 2$<br>F & H   |
| Mode                          | Most frequently occurring value F & H   |
| Range                         | Range = maximum - minimum F & H   |
| Interquartile Range           | $\text{IQR} = Q3 - Q1$<br>$Q1 = \text{lower quartile, } Q3 = \text{upper quartile}$<br>F & H                                      |
| Quartiles — Position          | $Q1$ at position $(n+1)/4$ ; $Q3$ at position $3(n+1)/4$<br>F & H   |
| Frequency Density (Histogram) | $\text{Frequency Density} = \text{Frequency} \div \text{Class Width}$<br>Higher   |
| Frequency from Histogram      | $\text{Frequency} = \text{Frequency Density} \times \text{Class Width}$<br>Higher   |
| Probability                   | $P(\text{event}) = \text{number of favourable outcomes} \div \text{total outcomes}$<br>F & H                                      |
| Complement                    | $P(A') = 1 - P(A)$ F & H  |
| AND Rule — Independent Events | $P(A \text{ and } B) = P(A) \times P(B)$ F & H  |
| OR Rule — Mutually Exclusive  | $P(A \text{ or } B) = P(A) + P(B)$ F & H  |
| Conditional Probability       | $P(A   B) = P(A \text{ and } B) \div P(B)$ Higher   |
| Expected Frequency            | $\text{expected} = P(\text{event}) \times \text{number of trials}$<br>F & H   |
| Relative Frequency            | $\text{relative frequency} = \text{frequency} \div \text{total trials}$<br>F & H  |
| Stratified Sampling           | $\text{sample from subgroup} = (\text{subgroup size} / \text{total}) \times \text{sample}$<br>Higher                              |
| Venn Diagrams                 | $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ F & H  |
| Tree Diagrams                 | Multiply along branches (AND), add between branches (OR) F & H  |
| Scatter Graphs — Correlation  | Positive: both increase.<br>Negative: one increases, other falls<br>No correlation: no pattern<br>F & H                           |
| Cumulative Frequency          | Running total of frequencies;<br>plot at upper class boundary<br>Higher   |
| Box Plot                      | Shows: min, Q1, median, Q3, max Higher  |

## EXAM DAY QUICK REFERENCE — MUST-KNOW FORMULAS

**Quadratic Formula**  
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Pythagoras**  
 $a^2 + b^2 = c^2$

**SOH-CAH-TOA**  
 $\sin = O/H$   $\cos = A/H$   $\tan = O/A$

**Sine Rule**  
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine Rule**  
 $a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$

**Area (trig)**  
 $\frac{1}{2}ab \sin(C)$

**Area of Circle**  
 $A = \pi r^2$

**Circumference**  
 $C = 2\pi r$

**Vol — Cone**  
 $V = \frac{1}{3}\pi r^2 h$

**Vol — Sphere**  
 $V = \frac{4}{3}\pi r^3$

**Vol — Cylinder**  
 $V = \pi r^2 h$

**Compound Interest**  
 $A = P(1 + r/100)^n$

**% Change**  
 $(\text{change} \div \text{original}) \times 100$

**Density**  
 $D = M \div V$

**Speed**  
 $S = D \div T$

**Pressure**  
 $P = F \div A$

**Angles in Polygon**  
 $(n-2) \times 180^\circ$

**Mean (grouped)**  
 $\frac{\sum fx}{\sum f}$

**Probability**  
 $\text{favourable} \div \text{total outcomes}$

**IQR**  
 $Q3 - Q1$

**Freq Density**  
 $\text{frequency} \div \text{class width}$

**Nth Term (linear)**  
 $dn + (a - d)$

**Reverse %**  
 $\text{final} \div \text{multiplier}$

**Scale Factor (area)**  
 $\text{area SF} = (\text{length SF})^2$

**Practise every one of these with instant AI examiner feedback**

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