GCSE MATHEMATICS · FORMULA & REVISION SHEET 1 OF 3

Number, Algebra & Ratio

Number, algebra, and the full Ratio/Proportion/Rates of Change strand — Foundation core with Higher-tier extensions clearly marked. Works for AQA, Edexcel, OCR and WJEC.

FOUND + HIGHER needed by everyone HIGHER Higher tier only

1 Number — Place Value, Operations & Types

Order of Operations — FOUND + HIGHER

BIDMAS / BODMAS

Brackets, Indices, Division & Multiplication, Addition & Subtraction

Work left to right for operations of equal priority

Types of Number — FOUND + HIGHER

Integers: *…, −2, −1, 0, 1, 2, …*

Prime: *exactly two factors (2, 3, 5, 7, 11…)*

2 is the only even prime

Rational: *can be written as a fraction*

Irrational: *e.g. √2, π — non-repeating decimals*

Prime Factors / HCF / LCM — FOUND + HIGHER

Write number as product of primes (factor tree)

HCF *= product of common prime factors*

LCM *= product using highest power of each prime*

HCF × LCM = a × b

for two numbers a and b

Rounding & Estimation — FOUND + HIGHER

Significant figures: *first non-zero digit counts first*

Estimate: round each value to 1 s.f. then calculate

Error interval: *value ± half the rounding unit*

e.g. x = 7 (nearest int) → 6.5 ≤ x < 7.5

Fractions — FOUND + HIGHER

Add/Subtract: *common denominator first*

Multiply: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

Divide: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

flip the second, then multiply

Percentages — FOUND + HIGHER

% of amount: *(% ÷ 100) × amount*

Multiplier: *+15% → ×1.15, −15% → ×0.85*

% change $= \frac{change}{original} \times 100$

Reverse %: *divide by the multiplier*

e.g. £60 after +20% → 60 ÷ 1.2 = £50

Compound Growth & Decay — FOUND + HIGHER

Amount = P × (multiplier)^n

P = start, n = number of periods

Growth +r%: multiplier $= 1 + \frac{r}{100}$

Decay −r%: multiplier $= 1 - \frac{r}{100}$

Standard Form — FOUND + HIGHER

A × 10^n *where 1 ≤ A < 10*

Large numbers: n positive. Small: n negative

Multiply: multiply A's, add powers of 10

Divide: divide A's, subtract powers of 10

Surds (manipulation)

HIGHER

$\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$

$\sqrt{a} \div \sqrt{b} = \sqrt{(a/b)}$

Rationalise: $\frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$

multiply top & bottom by √a

For $\frac{1}{a+\sqrt{b}}$*: multiply by* $(a-\sqrt{b})$

the conjugate

Recurring Decimals → Fractions

HIGHER

Let x = recurring decimal

Multiply by 10, 100… to shift one full block

Subtract to eliminate the recurring part, solve for x

e.g. $0.45\dot{} = \frac{45}{99} = \frac{5}{11}$

Index Laws

FOUND + HIGHER

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^0 = 1$

Negative & Fractional Indices

HIGHER

$a^{-n} = \frac{1}{a^n}$

$a^{1/n} = \sqrt[n]{a}$

$a^{m/n} = (\sqrt[n]{a})^m$

e.g. $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$

Bounds & Accuracy

HIGHER

Add / Multiply: *use upper + upper or lower + lower*

Subtract: *upper − lower (and lower − upper)*

Divide: $\frac{UB}{LB}$ *for max,* $\frac{LB}{UB}$ *for min*

UB = upper bound, LB = lower bound

Powers & Roots

FOUND + HIGHER

Square: $5^2 = 25$ · **Square root:** $\sqrt{25} = 5$

Cube: $2^3 = 8$ · **Cube root:** $\sqrt[3]{8} = 2$

Squaring and square-rooting are inverse operations

A square root has two values: $\sqrt{25} = \pm 5$ *when solving*

2 Ratio, Proportion & Rates of Change

Ratio

FOUND + HIGHER

Share in ratio a : b → total parts = a + b

Find value of one part = total ÷ total parts

Simplify ratios by dividing by the HCF

Convert ratio to fraction: $a : b \rightarrow \frac{a}{a+b}$ *of the whole*

Proportion

FOUND + HIGHER

Direct proportion: $y = kx$ $(y \propto x)$

Inverse proportion: $y = \frac{k}{x}$ $(y \propto \frac{1}{x})$

Method: *use given values to find k, then use the equation*

Higher forms: $y = kx^2$, $y = k\sqrt{x}$, $y = \frac{k}{x^2}$

Compound Measures

FOUND + HIGHER

Speed $= \frac{distance}{time}$

Density $= \frac{mass}{volume}$

Pressure $= \frac{force}{area}$

Rearrange each as a formula triangle; watch the units

Money & Best Buys

FOUND + HIGHER

Best buy: *compare price per unit (£ per gram / per ml)*

Currency: *multiply/divide by the exchange rate*

Simple interest $= \frac{P \times R \times T}{100}$

P = principal, R = rate %, T = years

Compound Interest & Depreciation

HIGHER

$$\text{Value} = P \times \left(1 + \frac{r}{100}\right)^n$$

growth / interest

$$\text{Value} = P \times \left(1 - \frac{r}{100}\right)^n$$

depreciation / decay

n = number of years/periods

Exponential Growth & Decay

HIGHER

General form: $y = k \cdot a^x$

$a > 1$ → growth · $0 < a < 1$ → decay

Used for population, radioactive decay, investments

3 Algebra — Expressions, Expanding & Factorising

Expanding Brackets

FOUND + HIGHER

$a(b + c) = ab + ac$

$(x + a)(x + b) = x^2 + (a+b)x + ab$

FOIL / grid method

Factorising

FOUND + HIGHER

Common factor: *$ab + ac = a(b + c)$*

Quadratic: *$x^2 + bx + c$ → two numbers that add to b, multiply to c*

Difference of squares: *$a^2 - b^2 = (a + b)(a - b)$*

Factorising harder quadratics

HIGHER

$ax^2 + bx + c$: split the middle term

Find two numbers multiplying to $a \times c$, adding to b

Factor by grouping

Completing the Square

HIGHER

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

Turning point of $y = (x + p)^2 + q$ is $(-p, q)$

Used to solve quadratics and find min/max

Algebraic Fractions

HIGHER

Factorise numerator & denominator, then cancel

Add/subtract: common denominator (as with numbers)

Multiply/divide as with ordinary fractions

Laws of Indices in Algebra

FOUND + HIGHER

Apply the same index laws to algebraic terms

e.g. $6x^5 \div 2x^2 = 3x^3$

$(2x^3)^4 = 16x^{12}$

4 Equations, Inequalities & Formulae

Linear Equations

FOUND + HIGHER

Do the same operation to both sides

Aim: isolate the variable

Check by substituting your answer back in

Rearranging Formulae

FOUND + HIGHER

Make a chosen variable the subject

Inverse operations, both sides

Higher: when the subject appears twice, factorise it out

Quadratic Formula

HIGHER

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

for $ax^2 + bx + c = 0$

Discriminant $b^2 - 4ac$:

> 0 → 2 real roots · $= 0$ → 1 repeated · < 0 → none

Solving Quadratics (methods)

FOUND + HIGHER

1. **Factorising** *(set each bracket = 0)*
2. **Quadratic formula** *(Higher)*
3. **Completing the square** *(Higher)*
4. **Graphically** *(where curve crosses x-axis)*

Simultaneous Equations

FOUND + HIGHER

Elimination: *make one variable match, add/subtract*

Substitution: *rearrange one, substitute into other*

Higher: one linear + one quadratic → substitute & solve

Inequalities

FOUND + HIGHER

Solve like equations…

…but **flip the sign** *when ×/÷ by a negative*

Represent on a number line: ○ open (< >), ● closed (≤ ≥)

Higher: quadratic inequalities — sketch & read regions

Iteration

HIGHER

Rearrange equation into x = g(x) form

$x_{n+1} = g(x_n)$

feed each answer back in

Repeat to converge on a root to required accuracy

Algebraic Proof

HIGHER

Even number = 2n · Odd number = 2n + 1

Consecutive integers: n, n+1, n+2 …

Manipulate algebraically to show the required result

5 Sequences, Graphs & Functions

Sequences

FOUND + HIGHER

Arithmetic nth term: $an + b$

a = common difference

Geometric: *multiply by a common ratio r; nth term* ar^{n-1}

Special: square (n^2), *cube* (n^3), *triangular* $\frac{n(n+1)}{2}$,

Fibonacci

Quadratic Sequences

HIGHER

nth term: $an^2 + bn + c$

2a = 2nd difference → *find a*

Use first terms to solve for b and c

Straight Line Graphs

FOUND + HIGHER

$y = mx + c$

m = gradient, c = y-intercept

Gradient m $= \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$

Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Length $= \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$

Pythagoras

Parallel & Perpendicular

HIGHER

Parallel: *same gradient m*

Perpendicular: *gradients multiply to −1*

Perp. gradient $= -\frac{1}{m}$

Graph Shapes to Recognise

FOUND + HIGHER

Linear: $y = mx + c$ *(straight)*

Quadratic: $y = ax^2$ *(parabola)*

Cubic: $y = ax^3$

Reciprocal: $y = \frac{k}{x}$ *(hyperbola)*

Exponential: $y = k \cdot a^x$ *(Higher)*

Function Notation

HIGHER

f(x) *means 'the function of x'*

fg(x) = f(g(x))

do g first, then f (composite)

Inverse $f^{-1}(x)$**:** *swap x and y, then rearrange*

Graph Transformations

HIGHER

y = f(x) + a: *shift up by a*

y = f(x + a): *shift left by a*

y = −f(x): *reflect in x-axis*

y = f(−x): *reflect in y-axis*

y = af(x): *stretch (vertical, factor a)*

y = f(ax): *stretch (horizontal, factor* $\frac{1}{a}$*)*

Real-Life & Kinematic Graphs

FOUND + HIGHER

Conversion graph: *straight line to convert units/currency*

Distance–time: *gradient = speed*

Velocity–time: *gradient = acceleration*

Area under v–t graph = distance

Higher: estimate area with trapezium rule; tangent gradient = rate

Exam technique — algebra: Always show every line of working on "solve" and "show that" questions: method marks are awarded for the steps, not just the final answer. When a quadratic won't factorise nicely, reach for the formula. Never divide both sides of an equation by a variable — you lose solutions (e.g. $x^2 = x$ has roots 0 *and* 1).

GCSE MATHEMATICS · FORMULA & REVISION SHEET 2 OF 3

Geometry & Measures

Area, volume, angles, Pythagoras, trigonometry, vectors and transformations — Foundation core with Higher-tier extensions clearly marked. Works for AQA, Edexcel, OCR and WJEC.

FOUND + HIGHER needed by everyone HIGHER Higher tier only

Note on what's "given" in the exam: On Edexcel & AQA Higher papers, the formula sheet now provides the **sine rule, cosine rule, area = ½ab sin C**, and the **volume/surface area of sphere & cone**. You should still learn them — looking them up wastes time. Everything else on these cards you are expected to **memorise**.

1 Perimeter, Area & Circles

Area of 2D Shapes — FOUND + HIGHER

Rectangle: *length × width*

Triangle: *½ × base × height*

Parallelogram: *base × height*

Trapezium: $\frac{1}{2}(a + b) \times h$

a, b = parallel sides

Circles — FOUND + HIGHER

Circumference = πd = 2πr

Area = πr²

d = diameter, r = radius

Arcs & Sectors — HIGHER

Arc length = $\frac{\theta}{360} \times \pi d$

θ = angle at centre

Sector area = $\frac{\theta}{360} \times \pi r^2$

Compound Shapes — FOUND + HIGHER

Split into simple shapes, find each area, add

Perimeter: add all outer edges only

2 Volume & Surface Area

Prisms & Cylinders — FOUND + HIGHER

Volume of prism = area of cross-section × length

Cylinder volume = πr²h

Cylinder surface area = 2πr² + 2πrh

Cuboid — FOUND + HIGHER

Volume = length × width × height

Surface area = 2(lw + lh + wh)

Sphere — HIGHER

Volume = $\frac{4}{3}\pi r^3$

Surface area = 4πr²

(Given on the Higher formula sheet)

Cone & Pyramid — HIGHER

Cone volume = $\frac{1}{3}\pi r^2 h$

Cone curved S.A. = πrl

l = slant height

Pyramid volume = $\frac{1}{3}$ **× base area × height**

3 Pythagoras & Trigonometry

Pythagoras' Theorem

FOUND + HIGHER

$a^2 + b^2 = c^2$

c = hypotenuse (longest side, opposite right angle)

Find a shorter side: $c^2 - b^2 = a^2$

Higher: works in 3D — apply twice

Trigonometry (right-angled)

FOUND + HIGHER

SOH: $\sin\theta = \frac{opp}{hyp}$

CAH: $\cos\theta = \frac{adj}{hyp}$

TOA: $\tan\theta = \frac{opp}{adj}$

Find an angle: use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$

Exact Trig Values

HIGHER

sin 0/30/45/60/90 = $0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$

cos *is the reverse order of sin*

tan 0/30/45/60 = $0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}$

tan 90 undefined

Sine & Cosine Rules

HIGHER

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area = $\frac{1}{2}ab\sin C$

(all given on the Higher sheet)

4 Angles, Bearings & Polygons

Angle Facts

FOUND + HIGHER

Angles on a straight line = **180°**

Angles around a point = **360°**

Angles in a triangle = **180°**

Angles in a quadrilateral = **360°**

Vertically opposite angles are equal

Parallel Line Angles

FOUND + HIGHER

Corresponding *(F-shape): equal*

Alternate *(Z-shape): equal*

Co-interior *(C-shape): add to 180°*

Polygons

FOUND + HIGHER

Sum of interior angles = (n − 2) × 180°

n = number of sides

Exterior angle = $\frac{360}{n}$ *(regular polygon)*

Interior + exterior = 180°

Bearings

FOUND + HIGHER

Measured **clockwise from North**

Always written with **3 figures** *(e.g. 075°)*

Back bearing: ± 180°

5 Circle Theorems (Higher)

Angle Theorems

HIGHER

Angle at **centre = 2 ×** *angle at circumference*

Angle in a **semicircle = 90°**

(Thales)

Angles in the **same segment** *are equal*

Opposite angles in a **cyclic quadrilateral** *add to 180°*

Tangent & Chord Theorems

HIGHER

A **tangent meets a radius at 90°**

Two tangents from a point are **equal in length**

Alternate segment theorem: *angle between tangent & chord = angle in alternate segment*

6 Transformations, Vectors & Constructions

Transformations

FOUND + HIGHER

Translation: *by a vector (x over y)*

Reflection: *needs a mirror line*

Rotation: *needs centre, angle, direction*

Enlargement: *needs centre & scale factor*

Enlargement & Similarity

HIGHER

Negative scale factor → other side of centre, inverted

Length *scale factor k*

Area *scale factor* = $\mathbf{k^2}$

Volume *scale factor* = $\mathbf{k^3}$

Vectors

HIGHER

Column vector: top = x movement, bottom = y

Add/subtract *component by component*

Magnitude = $\sqrt{(x^2 + y^2)}$

Parallel vectors are scalar multiples; prove points collinear via this

Constructions & Loci

FOUND + HIGHER

Perpendicular bisector: *equal arcs from both ends*

Angle bisector: *arcs from the vertex*

Locus from a point: *a circle*

Locus from a line: *parallel lines / 'racetrack'*

3D Shapes, Nets & Plans

FOUND + HIGHER

Net: *the 2D shape that folds into a 3D solid*

Plan: *the view from directly above*

Front / side elevation: *the view from the front / side*

Cross-section: *the 2D shape made by a straight cut*

Congruence & Proof

HIGHER

Two triangles are congruent if they match by:

SSS · **SAS** · **ASA** *(or AAS)* · **RHS**

State which condition you used to justify the proof

Congruent = identical; similar = same shape, scaled

Exam technique — geometry: For every angle answer, write the *reason* next to it ("alternate angles", "angle in a semicircle") — reasons carry marks. Check your calculator is in **degrees** mode before any trig. For bearings and trig in context, always draw and label a diagram first — most lost marks come from setting the triangle up wrong, not the calculation.

GCSE MATHEMATICS · FORMULA & REVISION SHEET 3 OF 3

Statistics & Probability

Averages, charts, probability rules, tree diagrams and the data-handling cycle — Foundation core with Higher-tier extensions clearly marked. Works for AQA, Edexcel, OCR and WJEC.

FOUND + HIGHER needed by everyone HIGHER Higher tier only

1 Averages & Spread

The Three Averages — FOUND + HIGHER

Mean = $\frac{\textit{sum of values}}{\textit{number of values}}$

Median = *middle value when ordered*
for n values, position = (n+1)/2

Mode = *most frequent value*

Range = highest − lowest
(a measure of spread, not an average)

Averages from Tables — FOUND + HIGHER

Mean from frequency table = $\frac{\Sigma(fx)}{\Sigma f}$

Grouped data: use **midpoints** *for x → estimated mean*

Modal class = *group with highest frequency*

Quartiles & IQR — HIGHER

Lower quartile (Q1): $\frac{1}{4}$ *of the way through*

Upper quartile (Q3): $\frac{3}{4}$ *of the way through*

Interquartile range = Q3 − Q1
smaller IQR = more consistent data

Comparing Distributions — FOUND + HIGHER

Compare an **average** *(typical value)…*

…and a **measure of spread** *(consistency)*

Always write comparisons **in context** *of the question*

2 Charts, Tables & Diagrams

Common Charts — FOUND + HIGHER

Bar chart: *discrete data, equal-width bars*

Pictogram: *uses a key/symbol*

Pie chart: *360° ÷ total = degrees per item*

Line graph: *trends over time*

Scatter Graphs — FOUND + HIGHER

Correlation: *positive, negative, or none*

Line of best fit: *straight line through the trend*

Interpolation *(within data) is reliable;* **extrapolation** *(beyond) is not*

Frequency Diagrams — FOUND + HIGHER

Frequency polygon: *plot midpoints, join with lines*

Two-way table: *rows & columns, totals must agree*

Stem-and-leaf: *ordered, with a key*

Histograms — HIGHER

Unequal class widths → use **frequency density**

Frequency density = $\frac{\textit{frequency}}{\textit{class width}}$

Frequency = frequency density × class width = *area of bar*

Cumulative Frequency

HIGHER

Running total of frequencies

Plot against the **upper** *class boundary*

Read off median (½n), Q1 (¼n), Q3 (¾n)

Use to draw a **box plot**

Box Plots

HIGHER

Show: min, Q1, median, Q3, max

Box spans Q1 → Q3 (the IQR)

Great for comparing two data sets at a glance

3 Probability

Probability Basics

FOUND + HIGHER

$$\text{P(event)} = \frac{\textit{favourable outcomes}}{\textit{total outcomes}}$$

All probabilities lie between **0 and 1**

P(not A) = 1 − P(A)

Probabilities of all outcomes sum to 1

Expected Frequency

FOUND + HIGHER

Expected number = P(event) × number of trials

$$\text{Relative frequency} = \frac{\textit{times it happened}}{\textit{total trials}}$$

experimental probability

Combined Events

FOUND + HIGHER

AND rule (independent): *P(A and B) = P(A) × P(B)*

OR rule (mutually exclusive): *P(A or B) = P(A) + P(B)*

General OR rule (Higher): *P(A or B) = P(A) + P(B) − P(A and B)*

Use a **sample space diagram** *for two events*

Tree Diagrams

FOUND + HIGHER

Multiply along *the branches (AND)*

Add *the relevant end results (OR)*

Each set of branches must sum to 1

Higher: 'without replacement' changes the 2nd probabilities

Venn Diagrams & Set Notation

HIGHER

A ∩ **B** = *intersection (in both)*

A ∪ **B** = *union (in either)*

A′ = *complement (not in A)*

ξ = *the universal set (everything)*

Conditional Probability

HIGHER

Probability of B **given** *A has happened*

Reduce the total to only the outcomes where A occurred

Common in 'without replacement' problems

4 Sampling & the Data Handling Cycle

Sampling

FOUND + HIGHER

Population: *the whole group studied*

Sample: *a smaller part used to represent it*

Random sample: *everyone equally likely to be chosen*

Bias: *a sample that isn't representative gives misleading results*

Bigger, more random samples → more reliable conclusions

Data Types

FOUND + HIGHER

Qualitative: *words / categories*

Quantitative: *numerical*

Discrete: *counted (whole values)*

Continuous: *measured (any value in a range)*

Capture–Recapture

HIGHER

Estimate a population size:

$$\frac{\textit{total population}}{\textit{1st sample marked}} = \frac{\textit{2nd sample size}}{\textit{marked in 2nd sample}}$$

Assumes population unchanged & mixing is random

Stratified Sampling

HIGHER

Sample reflects group proportions

$$\textbf{From each group} = \frac{\textit{group size}}{\textit{total}} \times \textbf{sample size}$$

Fairer than simple random when groups differ in size

Exam technique — statistics: For "compare the data" questions you must mention *both* an average and the spread, and phrase it in context (not "the mean is higher" but "on average Class A scored higher"). On tree-diagram and "without replacement" questions, write the second-stage fractions with the new denominator — that single step is where most marks are lost.