

Number, Algebra & Ratio

Number, algebra, and the full Ratio/Proportion/Rates of Change strand — Foundation core with Higher-tier extensions clearly marked. Works for AQA, Edexcel, OCR and WJEC.

FOUND + HIGHER needed by everyone **HIGHER** Higher tier only

1 Number — Place Value, Operations & Types

Order of Operations

FOUND + HIGHER

BIDMAS / BODMAS

Brackets, Indices, Division & Multiplication, Addition & Subtraction

Work left to right for operations of equal priority

Types of Number

FOUND + HIGHER

Integers: ..., -2, -1, 0, 1, 2, ...

Prime: exactly two factors (2, 3, 5, 7, 11...)

2 is the only even prime

Rational: can be written as a fraction

Irrational: e.g. $\sqrt{2}$, π — non-repeating decimals

Prime Factors / HCF / LCM

FOUND + HIGHER

Write number as product of primes (factor tree)

HCF = product of common prime factors

LCM = product using highest power of each prime

HCF × LCM = a × b

for two numbers a and b

Rounding & Estimation

FOUND + HIGHER

Significant figures: first non-zero digit counts first

Estimate: round each value to 1 s.f. then calculate

Error interval: value \pm half the rounding unit

e.g. $x = 7$ (nearest int) $\rightarrow 6.5 \leq x < 7.5$

Fractions

FOUND + HIGHER

Add/Subtract: common denominator first

Multiply: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

Divide: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

flip the second, then multiply

Percentages

FOUND + HIGHER

% of amount: $(\% \div 100) \times \text{amount}$

Multiplier: +15% $\rightarrow \times 1.15$, -15% $\rightarrow \times 0.85$

% change = $\frac{\text{change}}{\text{original}} \times 100$

Reverse %: divide by the multiplier

e.g. £60 after +20% $\rightarrow 60 \div 1.2 = \text{£}50$

Compound Growth & Decay

FOUND + HIGHER

Amount = P × (multiplier)ⁿ

P = start, n = number of periods

Growth +r%: multiplier = $1 + \frac{r}{100}$

Decay -r%: multiplier = $1 - \frac{r}{100}$

Standard Form

FOUND + HIGHER

$A \times 10^n$ where $1 \leq A < 10$

Large numbers: n positive. Small: n negative

Multiply: multiply A's, add powers of 10

Divide: divide A's, subtract powers of 10

Surds (manipulation)

HIGHER

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\sqrt{a} \div \sqrt{b} = \sqrt{a/b}$$

$$\text{Rationalise: } \frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

multiply top & bottom by \sqrt{a}

For $\frac{1}{a+\sqrt{b}}$: multiply by $(a-\sqrt{b})$
the conjugate

Recurring Decimals → Fractions

HIGHER

Let x = recurring decimal

Multiply by 10, 100... to shift one full block

Subtract to eliminate the recurring part, solve for x

$$\text{e.g. } 0.45 = \frac{45}{99} = \frac{5}{11}$$

Index Laws

FOUND + HIGHER

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

Negative & Fractional Indices

HIGHER

$$a^{-n} = \frac{1}{a^n}$$

$$a^{1/n} = \sqrt[n]{a}$$

$$a^{m/n} = (\sqrt[n]{a})^m$$

$$\text{e.g. } 27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$$

Bounds & Accuracy

HIGHER

Add / Multiply: use upper + upper or lower + lower

Subtract: upper - lower (and lower - upper)

Divide: $\frac{UB}{LB}$ for max, $\frac{LB}{UB}$ for min

UB = upper bound, LB = lower bound

Powers & Roots

FOUND + HIGHER

Square: $5^2 = 25$ · **Square root:** $\sqrt{25} = 5$

Cube: $2^3 = 8$ · **Cube root:** $\sqrt[3]{8} = 2$

Squaring and square-rooting are inverse operations

A square root has two values: $\sqrt{25} = \pm 5$ when solving

2 Ratio, Proportion & Rates of Change

Ratio

FOUND + HIGHER

Share in ratio $a : b \rightarrow$ total parts = $a + b$

Find value of one part = total \div total parts

Simplify ratios by dividing by the HCF

Convert ratio to fraction: $a : b \rightarrow \frac{a}{a+b}$ of the whole

Proportion

FOUND + HIGHER

Direct proportion: $y = kx$ ($y \propto x$)

Inverse proportion: $y = \frac{k}{x}$ ($y \propto \frac{1}{x}$)

Method: use given values to find k , then use the equation

Higher forms: $y = kx^2$, $y = k\sqrt{x}$, $y = \frac{k}{x^2}$

Compound Measures

FOUND + HIGHER

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{Pressure} = \frac{\text{force}}{\text{area}}$$

Rearrange each as a formula triangle; watch the units

Money & Best Buys

FOUND + HIGHER

Best buy: compare price per unit (£ per gram / per ml)

Currency: multiply/divide by the exchange rate

$$\text{Simple interest} = \frac{P \times R \times T}{100}$$

P = principal, R = rate %, T = years

Compound Interest & Depreciation

HIGHER

$$\text{Value} = P \times \left(1 + \frac{r}{100}\right)^n$$

growth / interest

$$\text{Value} = P \times \left(1 - \frac{r}{100}\right)^n$$

depreciation / decay

n = number of years/periods

Exponential Growth & Decay

HIGHER

$$\text{General form: } y = k \cdot a^x$$

$a > 1 \rightarrow$ growth $\cdot 0 < a < 1 \rightarrow$ decay

Used for population, radioactive decay, investments

3 Algebra — Expressions, Expanding & Factorising

Expanding Brackets

FOUND + HIGHER

$$a(b + c) = ab + ac$$

$$(x + a)(x + b) = x^2 + (a+b)x + ab$$

FOIL / grid method

Factorising

FOUND + HIGHER

$$\text{Common factor: } ab + ac = a(b + c)$$

Quadratic: $x^2 + bx + c \rightarrow$ two numbers that add to b , multiply to c

$$\text{Difference of squares: } a^2 - b^2 = (a + b)(a - b)$$

Factorising harder quadratics

HIGHER

$ax^2 + bx + c$: split the middle term

Find two numbers multiplying to $a \times c$, adding to b

Factor by grouping

Completing the Square

HIGHER

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

Turning point of $y = (x + p)^2 + q$ is $(-p, q)$

Used to solve quadratics and find min/max

Algebraic Fractions

HIGHER

Factorise numerator & denominator, then cancel

Add/subtract: common denominator (as with numbers)

Multiply/divide as with ordinary fractions

Laws of Indices in Algebra

FOUND + HIGHER

Apply the same index laws to algebraic terms

$$\text{e.g. } 6x^5 \div 2x^2 = 3x^3$$

$$(2x^3)^4 = 16x^{12}$$

4 Equations, Inequalities & Formulae

Linear Equations

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Do the same operation to both sides

Aim: isolate the variable

Check by substituting your answer back in

Rearranging Formulae

FOUND + HIGHER

Make a chosen variable the subject

Inverse operations, both sides

Higher: when the subject appears twice, factorise it out

Quadratic Formula

HIGHER

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

for $ax^2 + bx + c = 0$

Discriminant $b^2 - 4ac$:

$> 0 \rightarrow$ 2 real roots $\cdot = 0 \rightarrow$ 1 repeated $\cdot < 0 \rightarrow$ none

Solving Quadratics (methods)

FOUND + HIGHER

1. Factorising (set each bracket = 0)
2. Quadratic formula (Higher)
3. Completing the square (Higher)
4. Graphically (where curve crosses x-axis)

Simultaneous Equations

FOUND + HIGHER

Elimination: make one variable match, add/subtract

Substitution: rearrange one, substitute into other

Higher: one linear + one quadratic → substitute & solve

Inequalities

FOUND + HIGHER

Solve like equations...

...but **flip the sign** when \times/\div by a negative

Represent on a number line: ○ open ($< >$), ● closed ($\leq \geq$)

Higher: quadratic inequalities — sketch & read regions

Iteration

HIGHER

Rearrange equation into $x = g(x)$ form

$$x_{n+1} = g(x_n)$$

feed each answer back in

Repeat to converge on a root to required accuracy

Algebraic Proof

HIGHER

Even number = $2n$ · Odd number = $2n + 1$

Consecutive integers: $n, n+1, n+2 \dots$

Manipulate algebraically to show the required result

5 Sequences, Graphs & Functions

Sequences

FOUND + HIGHER

Arithmetic nth term: $an + b$

a = common difference

Geometric: multiply by a common ratio r ; n th term ar^{n-1}

Special: square (n^2), cube (n^3), triangular $\frac{n(n+1)}{2}$,

Fibonacci

Quadratic Sequences

HIGHER

n th term: $an^2 + bn + c$

$2a$ = 2nd difference → find a

Use first terms to solve for b and c

Straight Line Graphs

FOUND + HIGHER

$$y = mx + c$$

m = gradient, c = y -intercept

$$\text{Gradient } m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Length} = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

Pythagoras

Parallel & Perpendicular

HIGHER

Parallel: same gradient m

Perpendicular: gradients multiply to -1

$$\text{Perp. gradient} = -\frac{1}{m}$$

Graph Shapes to Recognise

FOUND + HIGHER

Linear: $y = mx + c$ (straight)

Quadratic: $y = ax^2$ (parabola)

Cubic: $y = ax^3$

Reciprocal: $y = \frac{k}{x}$ (hyperbola)

Exponential: $y = k \cdot a^x$ (Higher)

Function Notation

HIGHER

$f(x)$ means 'the function of x '

$$fg(x) = f(g(x))$$

do g first, then f (composite)

Inverse $f^{-1}(x)$: swap x and y , then rearrange

Graph Transformations

HIGHER

$y = f(x) + a$: shift up by a

$y = f(x + a)$: shift left by a

$y = -f(x)$: reflect in x -axis

$y = f(-x)$: reflect in y -axis

$y = af(x)$: stretch (vertical, factor a)

$y = f(ax)$: stretch (horizontal, factor $\frac{1}{a}$)

Real-Life & Kinematic Graphs

FOUND + HIGHER

Conversion graph: straight line to convert units/currency

Distance–time: gradient = speed

Velocity–time: gradient = acceleration

Area under v–t graph = distance

Higher: estimate area with trapezium rule; tangent gradient = rate

Exam technique — algebra: Always show every line of working on "solve" and "show that" questions: method marks are awarded for the steps, not just the final answer. When a quadratic won't factorise nicely, reach for the formula. Never divide both sides of an equation by a variable — you lose solutions (e.g. $x^2 = x$ has roots 0 and 1).